$$\Rightarrow w = \frac{-c}{12a\mu}\left[x^3 - 3xy^2 + 3a(x^2+y^2) - 4a^3\right]$$

$$\Rightarrow w = \frac{-c}{12a\mu}\left[(x-a)(x+2a-\sqrt{3}y)(x+2a+\sqrt{3}y)\right]$$

B. of the $w = 0$

$$(x-a)(x+2a-\sqrt{3}y)(x+2a+\sqrt{3}y) = 0$$

$$\Rightarrow x = a,\quad y = \frac{1}{\sqrt{3}}x + \frac{2a}{\sqrt{3}},\quad y = -\frac{1}{\sqrt{3}}x - \frac{2a}{\sqrt{3}}$$

which represent an equilateral triangle.

the flux Q of the fluid

$$Q = \iint w\,dx\,dy$$

$$= \frac{-c}{12a\mu}\int_{x=-2a}^{a}\int_{-\frac{x+2a}{\sqrt{3}}}^{x+2a/\sqrt{3}} (x^3 - 3xy^2 + 3ax^2 + 3ay^2 - 4a^3)\,dx\,dy$$

$$Q = \frac{-c}{6\sqrt{3}a\mu}\int_{-2a}^{a}\left\{(x^3+3ax^2-4a^3)(x+2a) - \frac{1}{3}(x-a)(x+2a)^3\right\}dx$$

$$Q = \frac{-c}{6\sqrt{3}a\mu}\left[\frac{2}{15}x^5 + \frac{10}{12}ax^4 + \frac{4a^2}{3}x^3 - \frac{4a^3}{3}x^2 - \frac{10}{3}x\right]_{-2a}^{a}$$

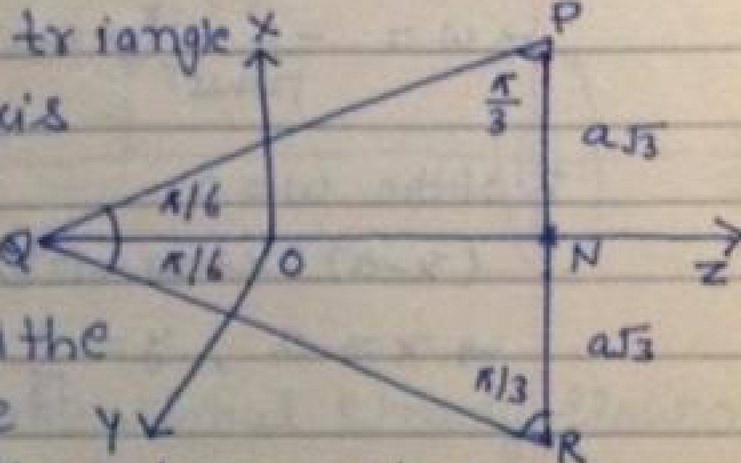
$$\boxed{Q = \frac{27}{2a\sqrt{3}}\,\frac{ca^4}{\mu}}$$

Average flux $= \frac{\text{flux}}{\text{Area}} = \frac{27}{2a\sqrt{3}}\,\frac{ca^4}{\mu}\,\frac{2}{3a\cdot 2a\sqrt{3}}$

$$\boxed{(Q)_{AF} = \frac{3ca^2}{2a\mu}}$$

Case III :- Steady flow in Pipes of equilateral Triangular section:-

Let each side of triangle be $2a\sqrt{3}$. the z-axis Passes through the centre of gravity of the section And the axis of x And y are Perpendicular to the two sides.



the equation to the boundary

$$(x-a)(x-\sqrt{3}y+2a)(x+\sqrt{3}y+2a)=0$$

the Laplace equation

$$w_1 = A(x^3-3xy^2)+B$$

Equilateral triangle section Assuming

$$w = A(x^3-3xy^2)+B-\frac{c}{4\mu}(x^2+y^2)$$

with the B.C. $w=0$

$$A(x^3-3xy^2)+B-\frac{c}{4\mu}(x^2+y^2)=0$$

since $x=a$ is Part of Boundary

$$A(a^3-3ay^2)+B-\frac{c}{4\mu}(a^2+y^2)=0$$

$$\Rightarrow Aa^3+B-\frac{c}{4\mu}a^2=0$$

And $-3Aa-\frac{c}{4\mu}=0$

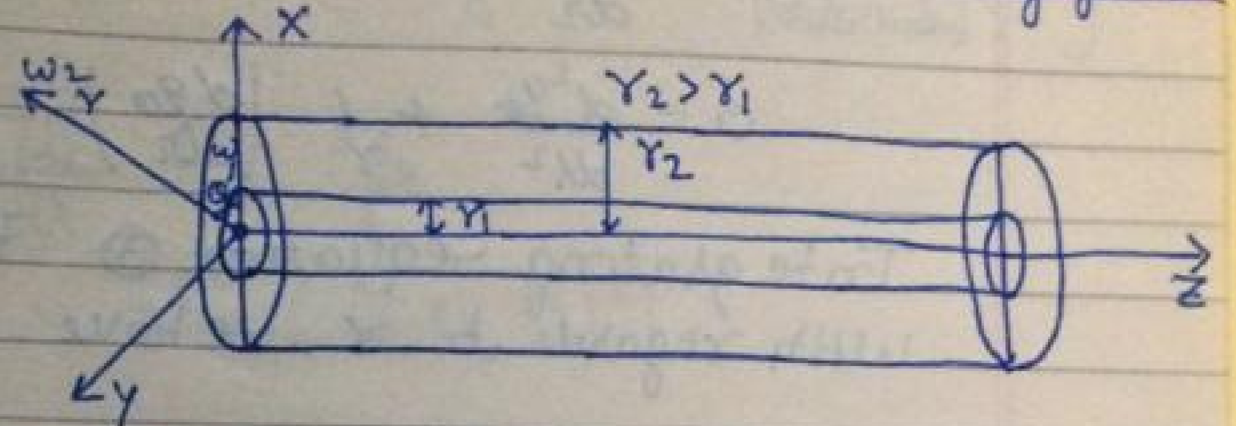
$$\Rightarrow A=\frac{-c}{12a\mu}, \quad B=\frac{ca^2}{3\mu}$$

thus

$$w=\frac{-c}{12a\mu}(x^3-3xy^2)+\frac{ca^2}{3\mu}-\frac{c}{4\mu}(x^2+y^2)$$

Imp

§ $\frac{16.4}{P\text{-}561}$ Laminar Flow between concentric Rotating cylinder



Consider the two-dimensional steady flow of an incompressible fluid between two concentric rotating cylinder of radius r_1 and r_2 ($r_2 > r_1$) Let z-axis is taken along the common axis of the cylinder and r denotes the radial direction measured outward from the z-axis the outward cylinder has a radius r_2 and it is rotating with angular velocity w_2 while the radius of the inner cylinder is r_1 and its angular velocity is ω_1

It follows that the flow be peripheral only.

$$q_r = 0 \ , \ q_z = 0$$

Equation of continuity

$$\frac{\partial q_\theta}{\partial \theta} = 0$$

$$\Rightarrow q_\theta = q_\theta(r)$$

Hence $\boxed{q_r = 0 \ , \ q_\theta = q_\theta(r) \ , \ q_z = 0}$

the only non-zero v. component is the tangential velocity q_θ which depend on equation of motion

$$\rho \frac{q_\theta^2}{r} = \frac{dP}{dr} \quad \text{——} \ (i)$$

Comparing

$$\frac{1}{B}\left(\frac{c}{4\mu}-A\right)=\frac{1}{a^2}$$

And $$\frac{1}{B}\left(\frac{c}{4\mu}+A\right)=\frac{1}{b^2}$$

$$\Rightarrow \quad A=\frac{c}{4\mu}\,\frac{a^2-b^2}{a^2+b^2}$$

And $$B=\frac{c}{2\mu}\,\frac{a^2b^2}{(a^2+b^2)}$$

Substituting thevalue of the constant in this Equation

$$w = A(x^2-y^2)+B-\frac{c}{4\mu}(x^2+y^2)$$

$$=\frac{c}{4\mu}\,\frac{a^2-b^2}{a^2+b^2}(x^2-y^2)+\frac{c}{2\mu}\,\frac{a^2b^2}{a^2+b^2}-\frac{c}{4\mu}(x^2+y^2)$$

$$\boxed{w=\frac{c}{2\mu}\,\frac{a^2b^2}{a^2+b^2}\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)}$$

the Flux $Q = \iint w\,dx\,dy$

$$=\frac{c}{2\mu}\,\frac{a^2b^2}{a^2+b^2}\iint\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)dx\,dy$$

$$\Rightarrow Q=\frac{c}{2\mu}\,\frac{a^2b^2}{a^2+b^2}\left[\iint dx\,dy-\frac{1}{a^2}\iint x^2\,dx\,dy\right.$$

$$\left.-\frac{1}{b^2}\iint y^2\,dx\,dy\right]$$

$$\Rightarrow Q=\frac{c}{2\mu}\,\frac{a^2b^2}{a^2+b^2}\left[\pi ab-\frac{1}{a^2}\pi ab\cdot\frac{a^2}{4}-\frac{1}{b^2}\pi ab\,\frac{b^2}{4}\right]$$

$$\Rightarrow \boxed{Q=\frac{\pi c}{4\mu}\,\frac{a^3b^3}{a^2+b^2}}$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{c}{\mu}$$

Boundary condition $w=0$ on the surface of the elliptic pipe

Consider the transformation

$$w = w_1 - \frac{c}{4\mu}(x^2+y^2)$$

where w_1 satisfies the equation

$$\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} = 0$$

with the boundary $w_1 = \frac{c}{4\mu}(x^2+y^2)$

Equation to the elliptic cross-section be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1$$

Let Solution of the Laplace equation is

$$w_1 = A(x^2-y^2) + B$$

For elliptic pipe Assuming

$$w = A(x^2-y^2) + B - \frac{c}{4\mu}(x^2+y^2)$$

on the boundary on the Pipe $w=0$

$$0 = A(x^2-y^2) + B - \frac{c}{4\mu}(x^2+y^2)$$

$$0 = \left(A - \frac{c}{4\mu}\right)x^2 - \left(A + \frac{c}{4\mu}\right)y^2 + B$$

$$\Rightarrow \frac{1}{B}\left(\frac{c}{4\mu} - A\right)x^2 + \frac{1}{B}\left(\frac{c}{4\mu} + A\right) = 1$$

this eq^n is identical to eq^n of the cross-section $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$